Fall 2001

Physics 129: Problem Set #5

Due: Wed Oct 10 at 5PM
Homework Box available on 2nd Floor LeConte breezeway

This problem set covers Perkins Section 3.12 and Sections 4.3 through 4.8

1. Perkins 3.8
2. Perkins 3.10
3. In class we saw that just as we can describe the angular momentum for a spin 1/2 particle with the 3 Pauli matrices, we can describe the SU(3) structure for a triplet state with eight 3×3 matrices which are labeled λ_1 through λ_8. The explict form of these matrices can be found in the Particle Data Book at the very end of the section called *SU(3) Isoscalar Factors anbd Represenation Matrices*.

   In this notation, the u, d and s are written:

$$
\begin{aligned}
u &= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \\
d &= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \\
s &= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}
\end{aligned}
$$

   (a) Show that the combinations $\lambda_1 \pm i\lambda_2$, $\lambda_4 \pm i\lambda_5$, and $\lambda_6 \pm i\lambda_7$ act as raising and lowering operators. In other words, they transform quarks of one species to quarks of another species.

   (b) The Δ^+ has a quark wave funtion $[uud + udu + duu]/\sqrt{3}$. Use one of the operators from part (a) to construct the state that corresponds to the Δ^0

   (c) Starting with the Δ^+, use one of the operators from part (a) to construct a state containing two up quarks and a strange quark.

4. The lowest lying pseudoscalar and vector multiplets are both SU(3)$_{flavor}$ nonets (octet plus singlet) with no orbital angular momentum. The mass splittings within a multiplet come from the difference in mass between the u, d and s quarks and to a smaller extent from electromagnetic corrections due to the differences in quark charges. Mass splitting between the pseudoscalar and vector multiplets is due to QCD hyperfine splitting. The hyperfine mass splitting for mesons with quark content $q_1\overline{q_2}$ means that an approximation to the meson mass is:

$$M(\text{meson}) = m_1 + m_2 + A\frac{\mathbf{S}_1 \cdot \mathbf{S}_2}{m_1 m_2} + E_{binding}$$

where A is a constant related to $|\psi(0)|^2$, $\mathbf{S}_i$ is the spin of quark i, m_i is the mass of quark i and $E_{binding}$ is the relevant strong potential binding energy ignoring the spin-spin term.

   (a) By comparing the masses of the K^0 and K^{0*} mesons, estimate the value of A

   (b) Use this value of A and the quark masses given in Perkins 4.37 to predict the splitting between the η and ω. Note: you may assume that the η is a pure SU(3) octet state (this is not completely true, but the mixing with the SU(3) singlet is small). In the case of the ω, the mixing is large; use the wave function given in Perkins 4.27.

5. The expression for the meson mass given in Problem 4 can be used for states involving heavy quarks.

   (a) Estimate the mass difference between the D^0 ($c\bar{u}$ pseudoscalar state) and the D^{0^*} the ($c\bar{u}$ vector state).

   (b) Estimate the mass difference between the D^0 and the $D_s{}^*$ the ($c\bar{s}$ vector state).

   You may assume a charm quark mass of 1.5 GeV. Compare your estimates to the particle data book values.